An inquiry-based geometry course for teacher education students

Jay Jahangiri, Mathematical Sciences, Kent State University, Ohio

jjahangi@kent.edu

Victor Oxman, Western Galilee College, Acre, Israel

Victor.oxman@gmail.com

Inquiry Based Teaching/Learning (IBTL)

- NCTM 2000:

  Principles and Standards for School Mathematics

- NCTM 2014:

  Principles to Actions

standards alone will not realize the goal of high levels of mathematical understanding by all students and so more is needed than standards (Linda M. Gojak KSU Graduate & NCTM President 2012-2014)

- NCTM 2020:

A natural and obvious focus was given to both pre-service and in-service teachers, who are in the forefront of the delivery of the standards.

Students' Reactions to IBTL

- Students' Resistance
- Low or Bad Teaching Evaluations
- If Probationary, don't try it
- If Tenured, why bother?

What If Not

- Problems Used
- Approach
- Implementation
- Two Examples

The WIN Problem Must

- Be of interest to the students.
- Have potential for generalization.
- Have interesting cases.
- Be challenging but not frustrating.
- Suitable for using Graphing Technology

Our Approach

- Start with the right problem
- Give students freedom to explore possible cases
- Stimulate Discussion

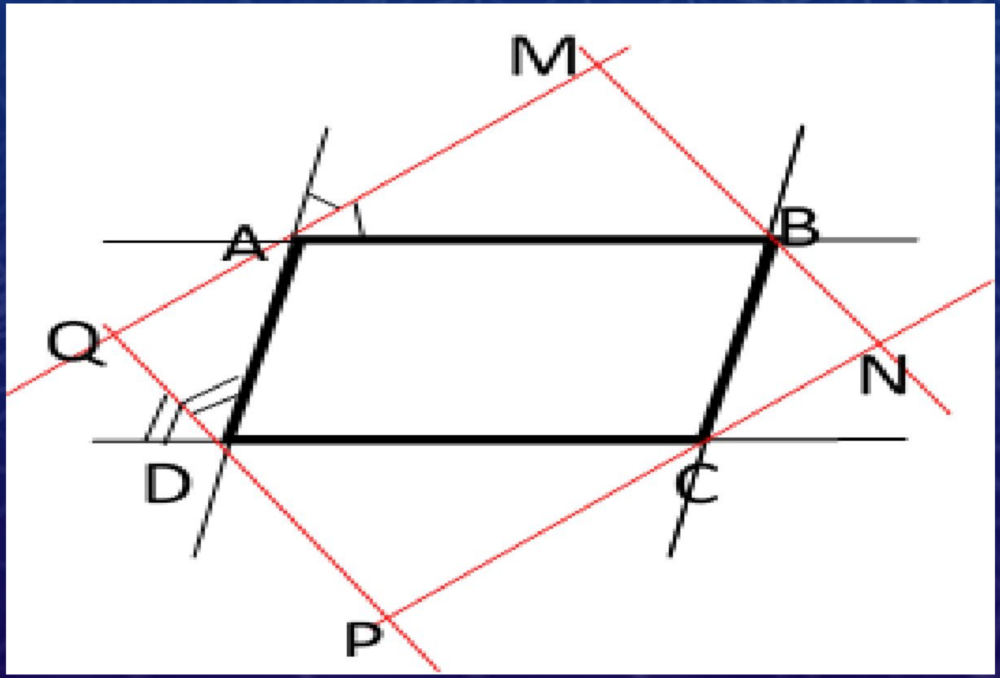
WIN Strategy Implementation

- What if more
- What if less
- What if instead

Example 1: ABCD is a parallelogram

MN, NP, PQ, QM are external angle biscetors.

Determine the shape of MNPQ



What if ABCD is:

- Square?
- Rectangle?
- Rhombus?
- Trapezoid (isosceles, scalene)?
- Arbitrary Quadrilateral?
- Not a quadrilateral (triangle, pentagon)?

What If:

- We bisect the interior angles instead?

- The angles are divided in ratios other than half-and-half?

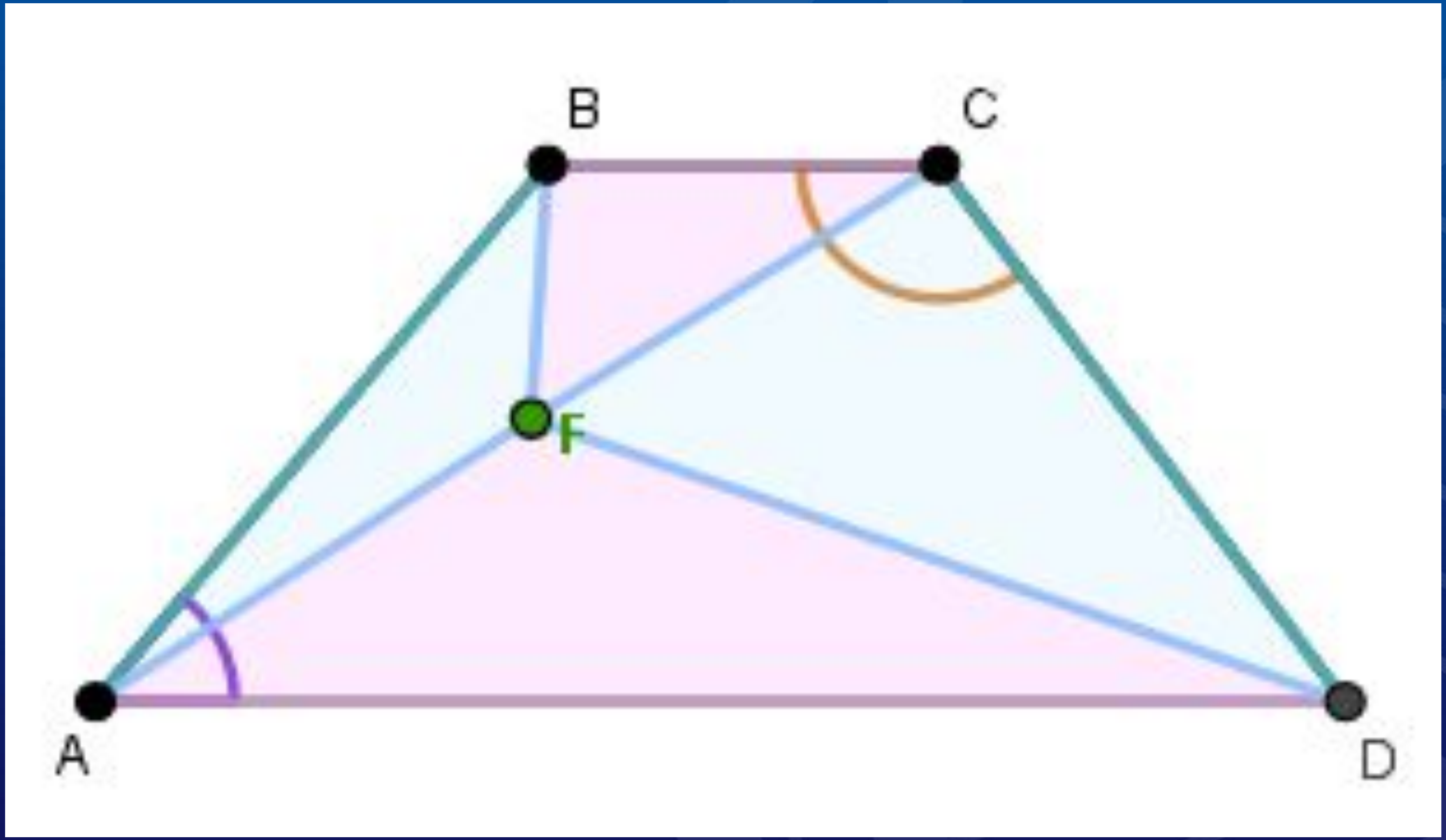
Table of Cases

Question	Strategy	Result
What if ABCD is a rectangle?	WIN	MNPQ is a square
What if ABCD is an isosceles trapezoid?	WIN	MNPQ is a right-angled kite
What if ABCD is a trapezoid?	WIN	MNPQ is a cyclic quadrilateral with two opposite right angles
What if ABCD has two equal opposite angles?	WIN	MNPQ is an isosceles trapezoid
What if ABCD is a rhombus?	WIN	MNPQ is a rectangle
What if ABCD is a cyclic quadrilateral?	WIN	MNPQ is a cyclic quadrilateral with perpendicular diagonals
What if MNPQ is an isosceles trapezoid?	WIN	ABCD is a quadrilateral with a pair of equal opposite angles

Example # 2

- **Given a quadrilateral ABCD whose sides AD and BC are parallel. We want to find out what is the set of points for which the sums of areas S_{BFC} + S_{AFD} and S_{AFB} + S_{DFC} are equal:**
- $S_{BFC} + S_{AFD} = S_{AFB} + S_{DFC} = (1/2)S_{ABCD}$

Example # 2



CASE 1

- Rectangle. Using the top two sliders and dragging the vertices of ABCD, you can get a rectangle. The applet allows the F point to automatically move inside the quadrilateral ABCD. To do this, just click the button in the lower left corner of the applet . When point F passes the position where PINK AREA = BLUE AREA, it leaves a trace of it on the screen in the form of a red dot. The applet allows you to discover that a set of such points forms the entire interior of the quadrilateral. This fact in itself is not complicated and can be easily explained by students.

CASE 2

- Parallelogram. This case is similar to the previous one and should also not present difficulties for students. Using the applet in a similar way, each can obtain a parallelogram of one form or another and discover that the required set of points is also the interior of the quadrilateral.

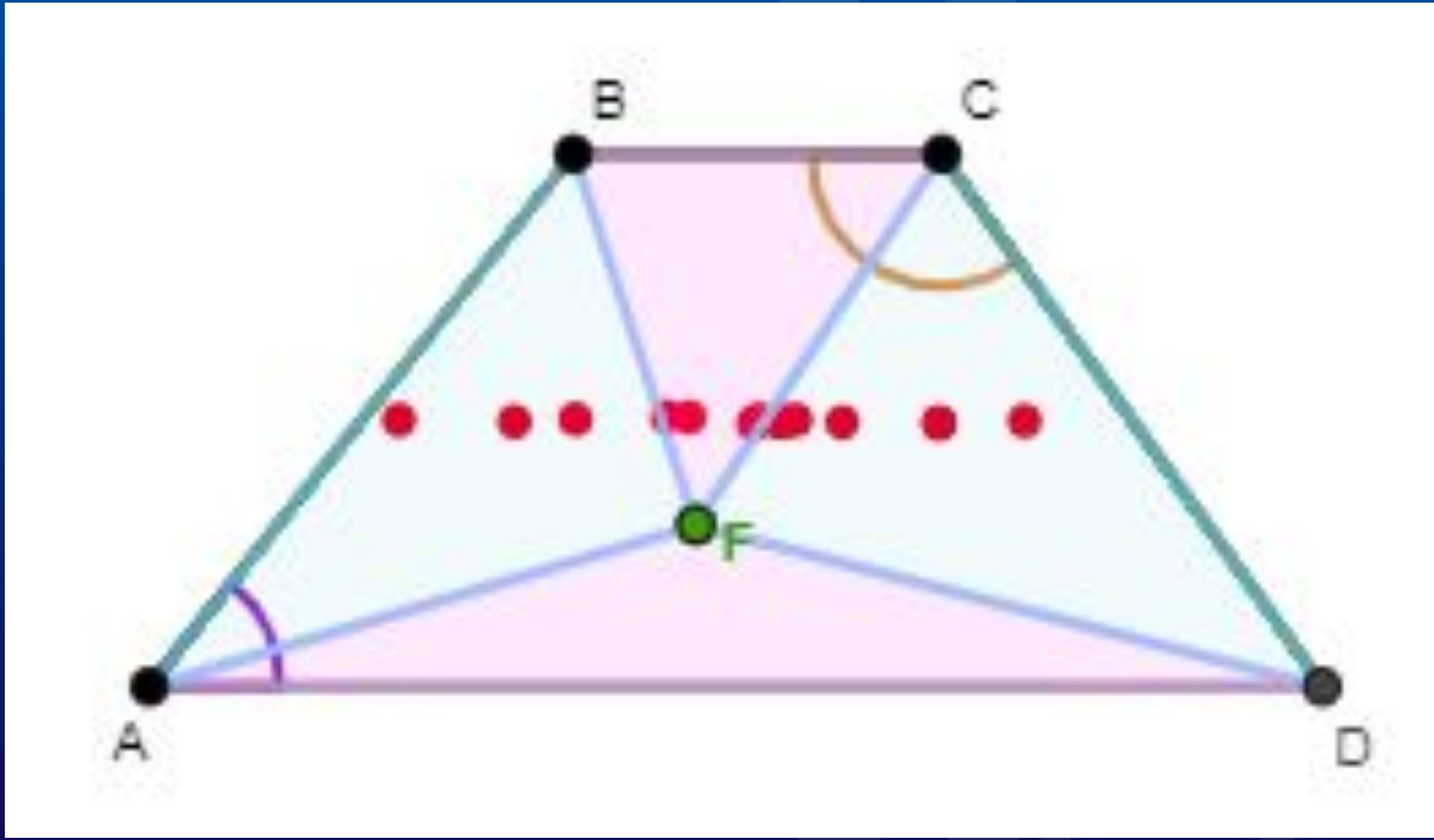
CASE 3

- Trapezoid. With the help of the top two sliders and dragging the vertices of ABCD one can obtain a trapezoid. By allowing point F to automatically move inside the trapezoid and observing all the red dots, one can discover after some time (when there are quite a lot of such points) that they form a segment parallel to the bases of the trapezoid. Upon closer examination, we can assume that this is the midline of the trapezoid. Proving this fact can also be done by students as a regular geometry problem.

CASE 4

- An arbitrary convex quadrilateral:
- Using the applet https://www.geogebra.org/m/vda3cxem one can observe that the red dots also line up in a straight line. Therefore, it is natural to assume that in the general case the locus inside a quadrilateral is a straight line segment. The button LOCUS allows you to verify this. Next, you can ask students to strictly prove this fact.

An interesting case



GeoGebra Applet # 1

The following cases are investigated using the applet

https://www.geogebra.org/m/yfnxfex2

GeoGebra Applet # 2

https://www.geogebra.org/m/vda3cxem

Q & A